Introduction to Formal Methods

Chapter 5. Timed Automata

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5. Timed Automata

- “Temporal”
  - “Trigger the alarm action upon detecting of a problem”

- “Real-Time”
  - “Trigger the alarm less than 5 seconds after detecting a problem”

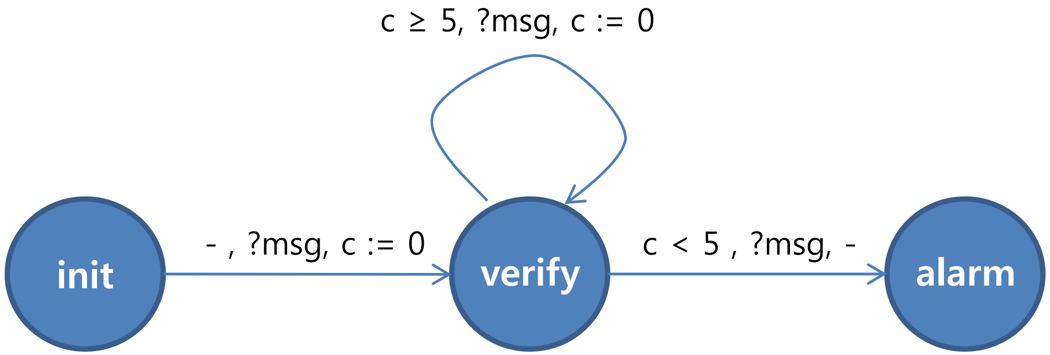
- Timed Automata
  - Proposed by Alur and Dill in 1994.
  - An answer to this “real-time” needs

- Organization of chapter 5
  - Description of a Timed Automata
  - Networks of Timed Automata and Synchronization
  - Variants and Extensions of the Basic Model
  - Timed Temporal Logic
  - Timed Model Checking

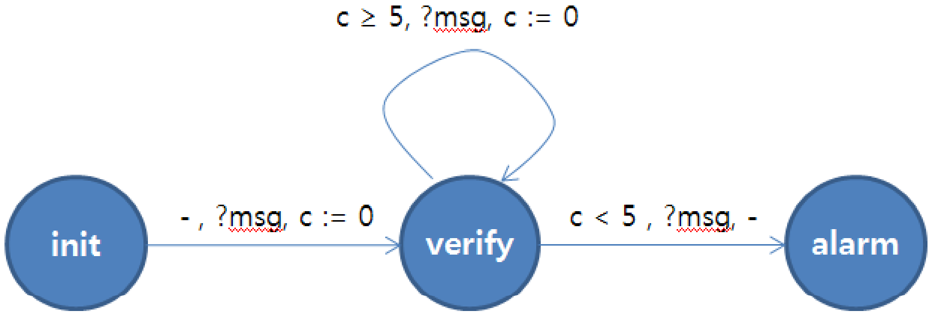
5.1 Description of Timed Automata

- Two fundamental elements of timed automata
  1. A finite automaton (assumed instantaneous between states)
  2. Clocks

- An example



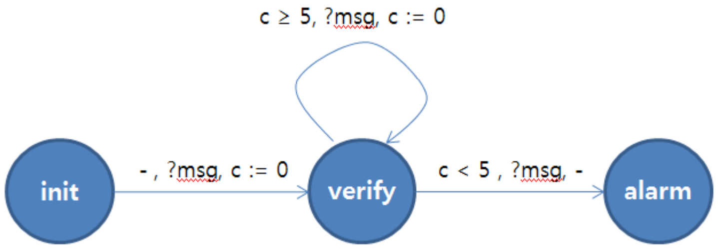
- Clocks and transitions
  - Clocks
    - Variables having non-negative real values in R
    - All clocks are null in the initial system states
    - All clocks evolve at the same speed, synchronously with time
  - Transitions
    - Three items
    - A guard
    - An action (label)
    - Reset of some clocks
  - The system operates as if equipped with
    - A global clock
    - Many individual clocks (each is synchronized with the global clock)



- Configurations and executions
  - Configuration of the system
    - (q, υ)
    - q : a current control state of the automaton
    - υ : the value of each clock

    - We also refer to υ as a valuation of the automaton clocks.
    - Time automata does not fix the time unit under consideration
  - Execution of the system
    - (usually infinite) sequence of configurations
    - A mapping ρ from R to the set of configuration

    - Configurations change in two ways
      - Delay transition
      - Discrete transition (or action transition)



Discrete transition

$$(\text{init}, 0) \rightarrow (\text{init}, 10.2) \xrightarrow{?\text{msg}} (\text{verify}, 0) \rightarrow (\text{verify}, 5.8) \xrightarrow{?\text{msg}} (\text{verify}, 0) \rightarrow (\text{verify}, 3.1) \xrightarrow{?\text{msg}} (\text{alarm}, 3.1) \rightarrow \ldots$$

Delay transition

- Trajectory
  - $\rho(0)$: the initial state
  - $\rho(12.3)$ = (verify, 2.1)

5.2 Networks of Timed Automata and Synchronization

- It is useful to build a timed model in a composite fashion,
  - by combining several parallel automata synchronized with one another
  - → a timed automata network

- Executions of a timed automata network
  - All automata components run in parallel at the same speed
  - Their clocks are all synchronized to the same global clock
  - (q, v) : a network configuration
    - q : a control state vector
    - v : a function associating with each network clock its value at the current time

- Synchronization
  - Timed automata synchronize on transitions (as usually) by resetting the clocks
  - The clocks which were not reset are unchanged
  - No concurrent write conflicts on clocks, since reset writes a zero value and nothing else

- Example : modeling a railroad crossing
  - Timed automata synchronize on transitions (as usually) by resetting the clocks
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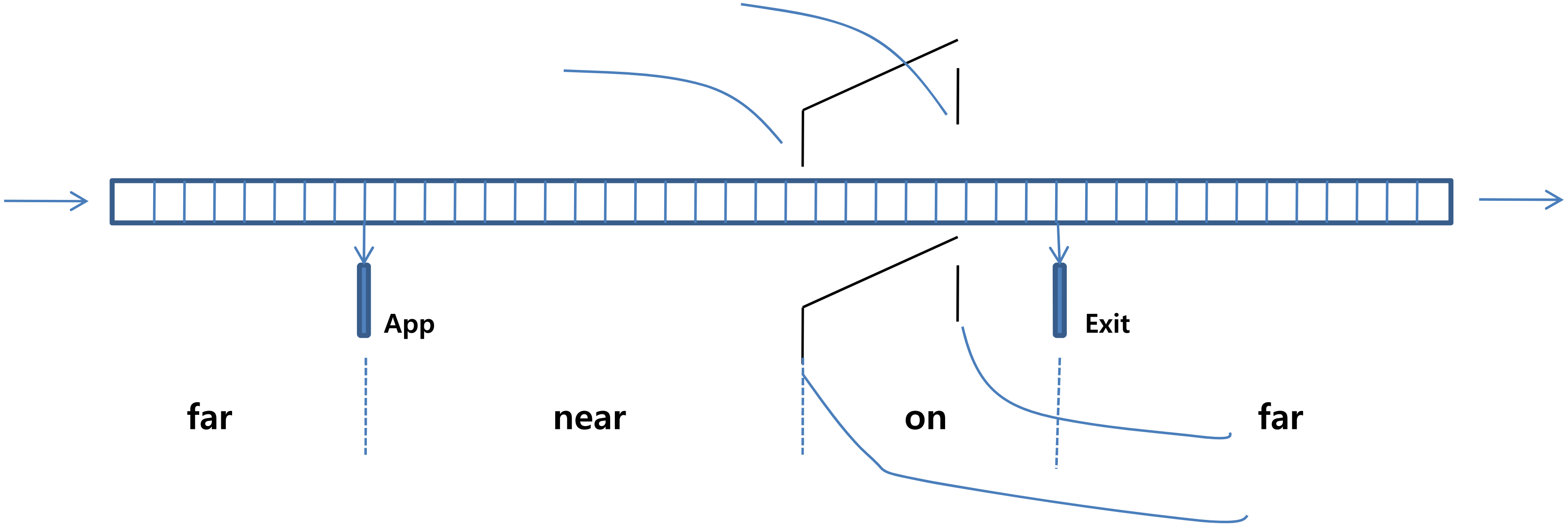
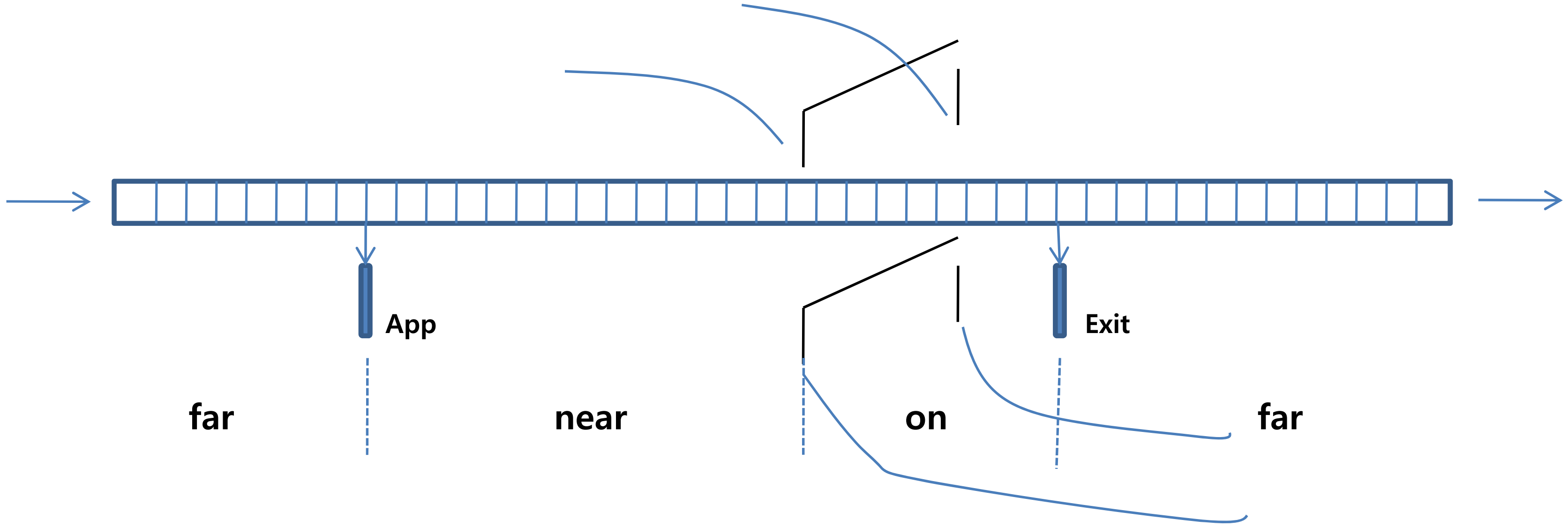
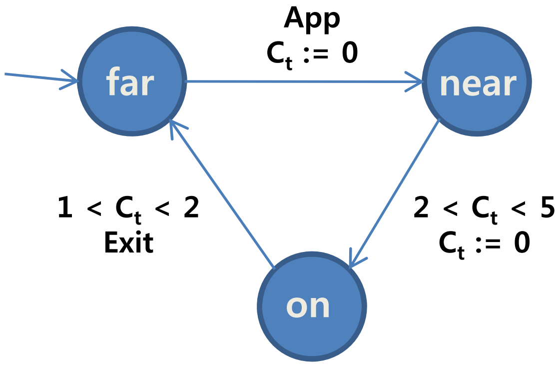
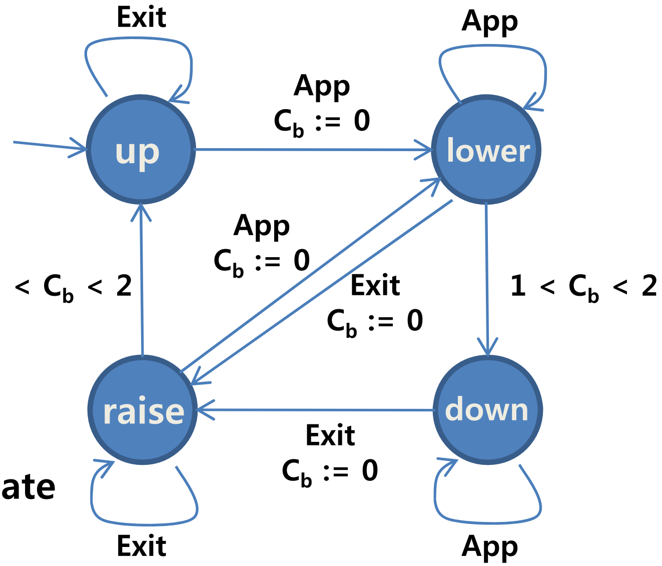



App
Exit
far
near
on
far


far
near
on
App
$C_t := 0$
$2 < C_t < 5$
$C_t := 0$
$1 < C_t < 2$
Exit

Train


up
lower
raise
down
Exit
App
App
$C_b := 0$
App
$C_b := 0$
Exit
$C_b := 0$
$1 < C_b < 2$
$1 < C_b < 2$
Exit
$C_b := 0$
Exit
App

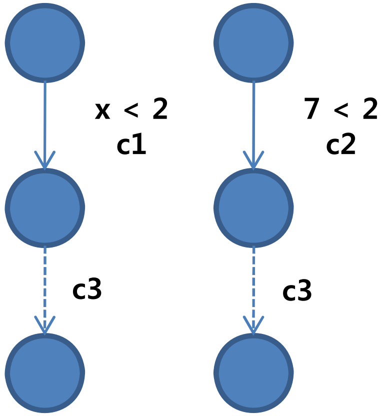
Gate

5.3 Variants and Extensions of the Basic Models

- Many variants, and three extensions

1. Invariants
   - Liveness hypothesis in the untimed model
   - Invariant: a state's condition on the clock values, which must always hold in the state
   - Example: near (invariant: $H_t < 5$), on (invariant: $H_t < 2$), $\mathrm{lower/raise}$ (invariant: $H_b < 2$)

2. Urgency
   - Used when cannot tolerate a time delay
   - Represented in the system configurations, not in the transitions
   - Allowing urgent/synchronized behaviors in a more natural way



3. Hybrid linear system
   - Models dynamic variables (in a form of differential equiations)
   - HYTECH

5.4 Timed Temporal Logic

- Given a system described as a network of timed automata,
- We wish to be able to state/verify properties of this system
  - Temporal properties
    - "When the train is inside the crossing, the gate is always closed."
  - Real-time properties
    - "The train always triggers an Exit signal within 7 minutes of having emitted an App signal."

- Three ways to formally state real-time properties
  1. Express it in terms of the reachability of some sets of configurations

  2. Use observer automata in PLTL model checking
     - Given a property ϕ, a network R
     - Testing reachability of some states in the product $R \parallel A_{\phi}$
     - UPPAAL , HYTECH

  3. Use a timed logic
     - TCTL (Timed CTL)
     - Etc.

- TCTL (Timed CTL)

  - $\Phi\ ,\ \Psi ::= P_1 \mid P_2 \mid \dots$ (atomic proposition)

    $\mid \neg\Phi \mid \Phi \wedge \Psi \mid \Phi \Rightarrow \Psi \mid \dots$ (boolean combinators)

    $\mid EF_{(\sim k)}\Phi \mid EG_{(\sim k)}\Phi \mid E\Phi\, U_{(\sim k)}\Psi$ (temporal combinators)

    $\mid AF_{(\sim k)}\Phi \mid AG_{(\sim k)}\Phi \mid A\Phi\, U_{(\sim k)}\Psi$ (path quantifiers)

  - ~ : any comparison symbol from {< , ≤ , = , ≥ , >}
  - k : any rational number from Q. (real number)
  - Operator X does not exist in TCTL

  - Example :
    - $AG\ (pb \Rightarrow AG_{(\leq 5)}\ alarm)$
      - "If a problem occurs, then the alarm will sound immediately and it will sound for at least 5 time units."
    - $AG\ (\neg far \Rightarrow AF_{(<7)}\ far)$
      - "When the train is located in the railway section between the two sensors App and Exit, it will leave this section before 7 time units."

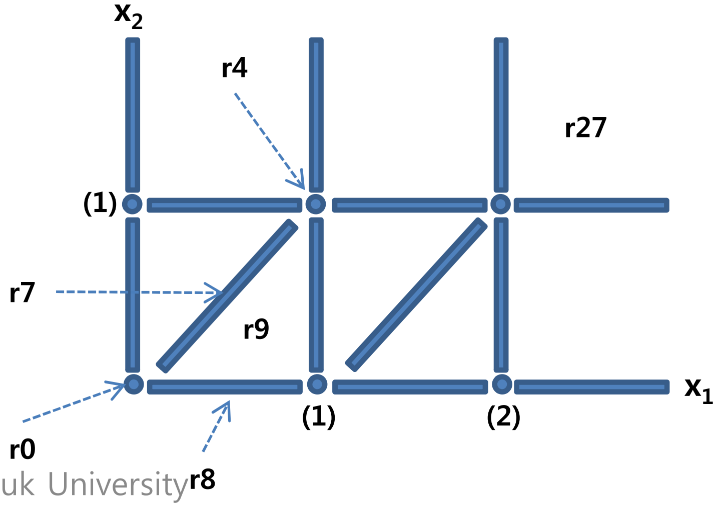
5.5 Timed Model Checking

- With timed automata and TCTL logic
- We wish to obtain a model checking algorithm for them.

- Difficulties : Automaton has an infinite number of configurations, since
  1. Clock values are unbounded
  2. The set of real numbers used in clocks is dense

  → Overcome it with the equivalence classes, called "*regions*"

  – Example: $x_1, x_2 \sim k$ with $k = 0, 1, 2$



- Complexity
  - Model checking algorithms are complicated.
  - The number of regions grows exponentially.

  - $O(n!M^n)$
    - n: number of clocks
    - M: upper bounds of every constant

  - No general and efficient method is likely to exist. (vs. linear complexity in CTL)
  - PSPACE-complete problem

  - Existing tools focus on defining adequate data structures for handing sets of regions → "*zones*"

  - Existing tools have been successfully used
    - HYTECH
    - KRONOS
    - UPPAAL

Conclusion of Part I

- Model checking is a verification technique
- It consists of three steps:
  1. Representation of a program or a system by an automaton
  2. Representation of a property by a logical formula
  3. Model checking algorithm
- Model checking is a powerful but restricted tool:
  - Powerfulness: exhaustive and automatic verification
  - Limitation: due to complexity barriers
  - In practice, the size of system is indeed the main obstacle yet to overcome.
- Model checker users are forced to simplify the model under analysis, until it is manageable. (Abstraction)