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Software Special Development 1

Introduction to Formal Methods

Part I : Formal Specification

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Reference

- " A Specifier's Introduction to Formal Methods "
  - Jeannette M. Wing, Carnegie Mellon University
  - IEEE COMPUTER, 1990

A Specifier's Introduction to Formal Methods

Jeannette M. Wing, Carnegie Mellon University

Formal methods used in developing computer systems are mathematically based techniques for describing system properties. Such formal methods provide frameworks within which people can specify, develop, and verify systems in a systematic, rather than ad hoc, manner.

A method is formal if it has a sound mathematical basis, typically given by a formal specification language. This basis provides the means of precisely defining notions like consistency and completeness and, more relevantly, specification, implementation, and correctness. It provides the means of proving that a specification is realizable, proving that a system has been implemented correctly, and proving properties of a system without necessarily running it to determine its behavior.

A formal method also addresses a number of pragmatic considerations: who uses it, what it is used for, when it is used, and how it is used. Most commonly, system designers use formal methods to specify a system's desired behavioral and structural properties.

However, anyone involved in any stage of system development can make use of formal methods. They can be used in the initial statement of a customer's requirements, through system design, implementation, testing, debugging, maintenance, verification, and evaluation.

Formal methods are used to reveal ambiguity, incompleteness, and inconsistency in a system. When used early in the system development process, they can reveal design flaws that otherwise might be discovered only during costly testing and debugging phases. When used later, they can help determine the correctness of a system implementation and the equivalence of different implementations.

Applied to computer systems development, formal methods provide mathematically based techniques that describe system properties. As such, they present a framework for systematically specifying, developing, and verifying systems.

For a method to be formal, it must have a well-defined mathematical basis. It need not address any pragmatic considerations, but lacking such considerations would render it useless. Hence, a formal method should possess a set of guidelines or a "style sheet" that tells the user the circumstances under which the method can and should be applied as well as how it can be applied most effectively.

One tangible product of applying a formal method is a formal specification. A specification serves as a contract, a valuable piece of documentation, and a means of communication among a client, a specifier, and an implementer. Because of their mathematical basis, formal specifications are more precise and usually more concise than informal ones.

Since a formal method is a method and not just a computer program or language, it may or may not have tool support. If the syntax of a formal method's specification language is made explicit, providing standard syntax analysis tools for formal specifications would be appropriate. If the language's semantics are sufficiently restricted, varying degrees of semantic analysis can be performed with machine aids as well. Thus, formal specifications have the additional advantage over informal ones of being amenable to machine analysis and manipulation.

For more on the benefits of formal specification, see Meyer.[1] For more on the distinction between a method and a language, and what specifying a computer system means, see Lamport.[2]

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Contents

Overview of Formal Methods

- Definition
- Features
- Applying Scope
- Pragmatic Considerations

Definition

- **Formal Methods**
  - Mathematically based techniques for describing system properties
    - Have a sound mathematical basis
    - Typically given by a formal specification language
  - Provide frameworks for systematically
    - Specifying,
    - Developing, and
    - Verifying systems

Features

- Formal methods provide means of precisely defining notions like
  - Completeness
  - Consistency
  - Specification
  - Implementation
  - Correctness

- Formal methods address a number of pragmatic considerations
  - Who
  - What
  - When
  - How it is used?
  - ex) System designers use a formal method to specify a system's desired behavioral and structural properties.

Applying Scope

- Any stage of system development can make use of formal methods
  1. Initial statement of a customer's requirements
  2. System design
  3. Implementation
  4. Testing
  5. Debugging
  6. Maintenance
  7. Verification
  8. Evaluation

- When used early,
  - Can reveal design flaws
- When used later,
  - Can help determine the correctness of a system implementation
  - Can help determine the equivalence of different implementations

Pragmatic Considerations

- Pragmatic considerations
  - A set of guidelines
  - Formal methods should tell the user
    1. Circumstances under which the method should and can be applied
    2. How it can be applied most effectively

- Formal Specification
  - One tangible product of applying formal methods
  - More precise and concise than informal specifications
  - A formal method's specification language may have Tool Supports
    1. Syntax analysis
    2. Semantic analysis with machine aids

> Formal Specification :
> Use mathematics to specify the desired properties of a computer system with support of automatic tools

Formal Specification Language

- Definition
- Syntactic Domains
- Semantics Domains
- Satisfies Relation
- Properties of Specifications
- Proving Properties of Specificands

Definition

- **Formal specification language**:

  $< Syn, Sem, Sat >$, where
  - *Syn* : syntactic domain
  - *Sem* : semantic domain
  - *Sat* : $Sat \subseteq Syn \times Sem$
    - *syn* is a specification of *sem*
    - *sem* is a specificand of *syn*

- Considerations
  - In principle, a formal method is based on some well-defined formal specification language
  - Formal specification language provides a formal method's mathematical basis
  - Formal methods differ because their specification languages have different syntactic and/or semantic domains

Syntactic Domains

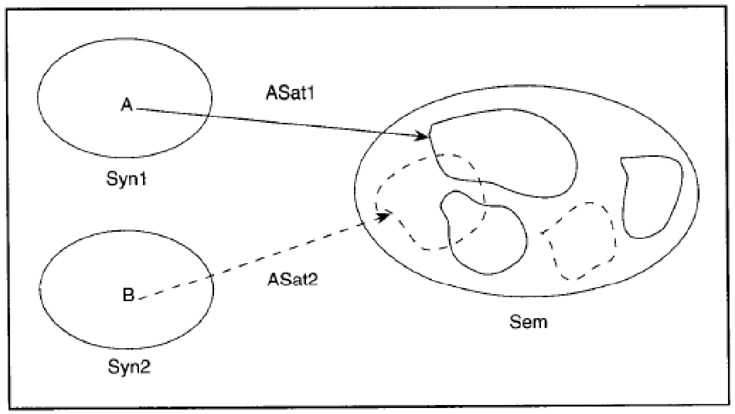
- *Syn*
  - a set of symbols
    - Constants
    - Variables
    - Logical connectives
  - a set of grammatical rules for combining symbols into well-formed sentences (semantics)
    - Ex) $\forall x.P(x) \Rightarrow Q(x)$: correct!!
      $\forall x. \Rightarrow P(x) \Rightarrow Q(x)$: wrong!!
  - Visual Specification : Graphical elements are also available
    - boxes, circles
    - lines, arrows
  - called **Specification**

Semantic Domains

- *Sem*
  - Formal specification languages differ most in their choice of semantic domains (Specificand) such as:
    - Abstract-data-type specification languages
      - algebra, theory, program
    - Concurrent and distributed systems specification languages
      - state sequence, event sequence, state and transition sequence
      - stream, synchronization tree, partial order
      - state machine
    - Programming languages
      - function from input to output, computation
      - predicate transformation
      - relation, machine instruction
      - called **Implementation**

Satisfies Relation

- *Sat*
  - Specifies different aspects of a single specificand using different specification languages:
    1. Behavioral specification aspect
       - Constraints on observable behavior of specificands
       - System's required functionality (mapping from inputs to outputs)
       - Others: fault tolerance, safety, security, response time, space efficiency
    2. Structural specification aspect
       - Constraints on the internal composition of specificands
       - Various hierarchical and uses relations
       - Call graph, data-dependency diagram, definition-use chain



Properties of Specifications

- Specification language should be defined as
  1. Unambiguous
     - If and only if it has exactly one meaning
     - Any natural languages and graphs are not formal inherently
  2. Consistent
     - If and only if its specificand set is non-empty
     - Cannot derive anything contradictory from the specification
     - There is some implementation that will satisfy the specification
  3. Complete
     - Need not be complete in the sense used in mathematical logic
     - Relatively-completeness properties might be desirable
     - In practice, we must usually deal with incomplete specifications

- A specification has **implementation bias** if it places unnecessary constraints on its specificand

Proving Properties of Specifications

- Most formal specification languages have logical inference systems
  - Can prove properties from the specification about specificands
  - Can predict system's behavior without executing or building the system
  - Can be mechanized
    - Theorem proving
    - Model checking
    - called **Formal Verification (Part II)**

Pragmatics

Users
Uses
Characteristics

Users

- 5 kind of users
  1. Specifier : write, evaluate, analyze, and refine specifications
  2. Customer : hired the specifiers
  3. Implementer : realize a specification
  4. Client : use a specified system
  5. Verifier : prove the correctness of implementations

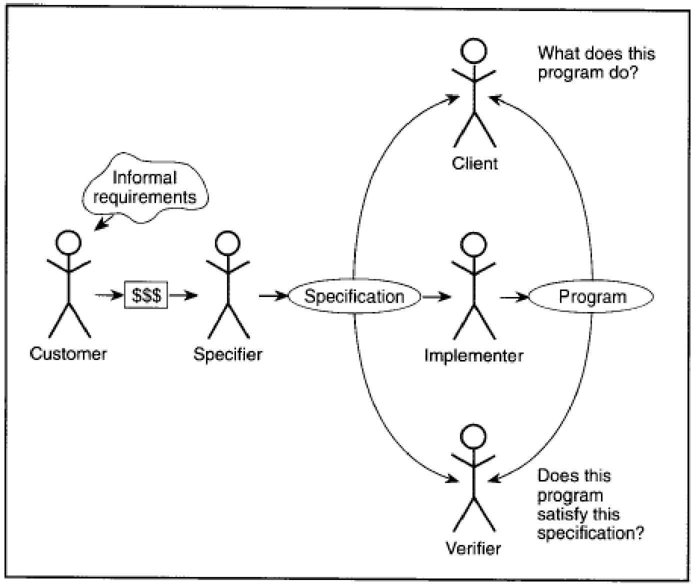


Figure 2. Specification users.

- A formal method's guidelines should identify
  1. Different types of users the method is targeted for
  2. Capabilities the users should have
  3. Application domain of the method

Uses

- The greatest benefit comes
  - from the process of formalizing
  - rather than the end result

- Can apply formal methods in all phases of SW development
  1. Requirements analysis
  2. System design
  3. System verification
  4. System validation
  5. System documentation
  6. System analysis and evaluation

- These applications should be considered as an integral one, **framework**

Uses 1. Requirements Analysis

- Formal methods help clarify customer's informally stated requirements
  - Crystallize customer's vague ideas
  - Reveal
    - Contradictions,
    - Ambiguities, and
    - Incompleteness in the requirements

- On the specification, both customers and specifiers can see
  - Whether it reflects customer's intuition
  - Whether specificand set has desired set of properties

Uses 2. System Design

- Two important activities during design
  1. Decomposition
  2. Refinement

- Decomposition
  - Process of partitioning a system into smaller modules
  - Interface specifications specify interfaces between modules

- Refinement
  - Process of refining modules at one level to modules at a lower level
  - Each refinement step should prove that a specification(program) at one level satisfies a higher level specifications
    - Program transformation, Program synthesis, Inferential programming
  - Formal methods and formal specification languages can state proof obligations(assumptions) precisely

Uses 3. System Verification

- System verification
  - Showing that a system satisfies its specification

- Formal Verification
  - Using formal specifications to verify a system
  - Cannot completely verify an entire system,
  - But can certainly verify smaller and critical part of system.
    - Gypsy, HDM(Hierarchical Development Method), FDM(Formal Development Method)
    - M-EVES(Environment for Verifying and Emulating Software)
    - HOL(Higher Order Logic)

- Difficulties in formal system verification
  - Should state explicitly assumptions about its environment : Not easy!
  - "Bounds of Formal Methods"

Uses 4. System Validation

- Formal methods can aid in system testing and debugging

- Specification alone :
  - Used to generate test cases for black-box testing
  - For boundary condition tests

- Specification along with implementation
  - Used to generate test cases
  - Additionally, can be used for testing analysis
    - Path testing
    - Unit testing
    - Integration testing
    - Etc.

Uses 5. System Documentation

- Formal specification
  - Captures “What” rather than “How”
  - Serves as a communication medium between
    - Clients and Specifiers
    - Specifiers and Implementers
    - Among members of an implementation team

Uses 6. System Analysis and Evaluation

- System analysis and evaluation
  - After system has been built and tested,
  - Critical analysis of its functionality and performance should be done
    - Does the system do what the customer wants?
    - Does it do it fast enough?
  - Formal method used in the development can help formulate and answer these questions

- Most formal methods have not yet been applied to specifying large-scale software and hardware systems
  - Size of the specification
  - Complexity of the specificand
    - Internal complexity
    - Interface complexity

Characteristics

- Formal method's characteristics influence the style in which a user applies it
  - Whether its language is graphical or textual
  - Whether its underlying logic is first-order or high-order
  - Etc.

- Formal method reflects a combination of many different characteristics:
  1. Model-oriented vs. Property-oriented
  2. Visual languages
  3. Executable
  4. Tool-supported

Characteristics 1. Model-oriented vs. Property-oriented

- Model-oriented methods
  - Define system's behavior directly by constructing a model of the system
  1. For sequential systems
     - Parnas' statemechines, VDM, Z, SCR, NuSCR
  2. For concurrent and distributed systems
     - Petri Nets, CCS, Hoare's CSP, Unity, I/O automata
     - Temporal logic, Lamport's transition axiom method, LOTOS

- Property-oriented methods
  - Define system's behavior indirectly by stating a set of properties using axioms
  1. Axiomatic methods
     - Iota, OBJ, Anna, Larch
  2. Algebraic methods
     - Act One

Algebraic specification of abstract data types can handle :
- Error values
- Nondeterminism
- parameterization

Characteristics 2. Visual Languages

- Visual specification languages
  - Any one who contains graphical elements in their syntactic domains

- Many examples
  - Petri nets : for concurrent systems
  - Statecharts : for specifying state transitions in reactive systems

- Semiformal methods
  - Multiple interpretations or text attached
  - Jackson's method (UML)
  - SASD, OOD
  - Requirements Engineering Methodology

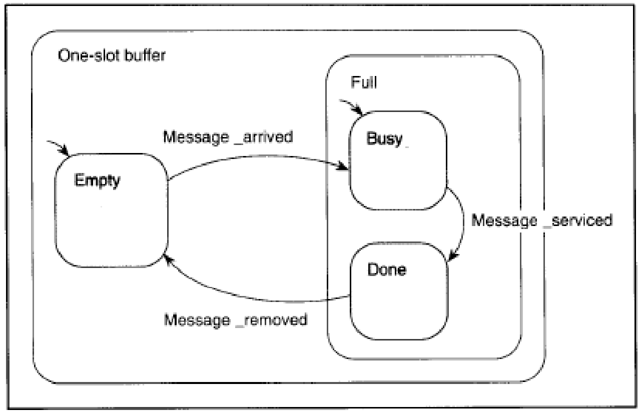


Figure 3. State chart specification of a one-slot buffer.

Characteristics 3. Executable

- Executable Specification
  - Can run on a computer

- Specifiers can use executable specifications
  - To gain immediate feedback about the specification itself.
  - To do rapid prototyping
  - To test a specificand through symbolic execution of the specification

- Many examples
  - Statecharts
  - OBJ
  - Prolog, Paisley
  - Most recent ones

Characteristics 4. Tool-supported

- Model-Checking tools
  - Let users construct a finite-state model of the system
  - Then show a property holds in each state or state transition of the system
  - EMC, SMV, SPIN

- Proof-checking tools
  - Let users treat algebraic specifications as rewrite rules
    - Larch Prover, Affirm, Reve
  - Handling first-order logic
    - Boyer-Moore Theorem Prover, FDM, HDM, m-EVES
  - Handling higher-order logic
    - HOL, LCF, OBJ

Some Examples

Abstract Data Type: Z, VDM, Larch
Concurrency: Temporal Logic, CSP, Transition Axioms

Some Examples

- 6 well-known formal methods (in 1990s)
  - Abstract data type : Z, VDM, Larch
    - Symbol table example
  - Concurrency : Temporal Logic, CSP, Transition Axioms
    - Unbounded buffer example

- When specifying the same problem with different methods, they look
  - Remarkably similar
  - Or totally different
  - Due to
    - Nature of the specificand
    - Simplicity of the specificand
    - Methods themselves

Abstract Data Type: Z, VDM, Larch

- 3 different specifications for a **symbol table**

Z

```
ST = KEY ⇸ VAL

INIT
  st′ : ST
  ──────────
  st′ = { }

INSERT
  st, st′ : ST
  k : KEY
  v : VAL
  ──────────
  k ∉ dom(st) ∧
  st′ = st ∪ {k ↦ v}

LOOKUP
  st, st′ : ST
  k : KEY
  v′ : VAL
  ──────────
  k ∈ dom(st) ∧
  v′ = st(k) ∧
  st′ = st

DELETE
  st, st′ : ST
  k : KEY
  ──────────
  k ∈ dom(st) ∧
  st′ = {k} ⩤ st
```

Figure 4. Z specification of a symbol table.

VDM

```
ST = map Key to Val

INIT()
ext  wr st : ST
post st′ = { }

INSERT( k : Key, v : Val)
ext  wr st : ST
pre  k ∉ dom st
post st′ = st ∪ {k ↦ v}

LOOKUP( k: Key)v : Val
ext  rd st : ST
pre  k ∈ dom st
post v′ = st(k)

DELETE( k : Key)
ext  wr st : ST
pre  k ∈ dom st
post st′ = {k} ⩤ st
```

Figure 5. VDM specification of a symbol table.

Larch

```
symbol_table is data type based on S from SymTab

  init = proc () returns (s: symbol_table)
    ensures s′ = emp ∧ new (s)

  insert = proc (s: symbol_table,k: key,v: val)
    requires ~ isin(s,k)
    modifies (s)
    ensures s′ = add(s,k,v)

  lookup = proc (s: symbol_table,k : key) returns (v: val)
    requires isin(s, k)
    ensures v′= find(s,k)

  delete = proc (s: symbol_table, k : key)
    requires isin(s,k)
    modifies (s)
    ensures s′ = rem(s,k)

  end symbol_table

SymTab: trait
  introduces
    emp: → S
    add: S,K,V → S
    rem: S,K → S
    find: S,K → V
    isin: S,K → Bool

  asserts
    S generated by (emp, add)
    S partitioned by (find, isin)
    for all (s: S,k,k1 : K,v : V)
      rem(add(s,k,v),k1) == if k = k1 then s else add(rem(s,k1),k,v)
      find(add(s,k,v),k1) == if k = k1 then v else find(s,k1)
      isin(emp,k) == false
      isin(add(s,k,v),k1) == (k = k1) ∨ isin(s,k1)

  implies
    converts (rem,find,isin) exempting (rem(emp),find(emp))
  end SymTab
```

Figure 6. Larch specification of a symbol table.

Abstract Data Type: Z, VDM, Larch

	Z (1988)	**VDM** (1986)	**Larch** (1985)
Base	Model-oriented (Also property-oriented)	Model-oriented	Property-oriented
Readability	Good	Normal	Bad
Specifiability	Bad	Normal	Good
Size	Normal	Compact	Long
Tool-Support	Proof Checker B	N/A	Syntax Analyzer Larch Prover

Concurrency: Temporal Logic, CSP, Transition Axioms

- 3 different specifications for an **unbounded buffer**

Temporal Logic

$$\langle \text{right!}m \rangle \Rightarrow \diamond\!\!\!-\, \langle \text{left!}m \rangle \quad (1)$$

$$(\langle \text{right!}m \rangle \wedge \ominus \diamond\!\!\!-\, \langle \text{right!}m' \rangle) \Rightarrow \diamond\!\!\!-\, (\langle \text{left!}m \rangle \wedge \ominus \diamond\!\!\!-\, \langle \text{left!}m' \rangle) \quad (2)$$

$$(\langle \text{left!}m \rangle \wedge \ominus \diamond\!\!\!-\, \langle \text{left!}m' \rangle) \Rightarrow (m \neq m') \quad (3)$$

$$(\langle \text{left!}m \rangle) \Rightarrow \diamond (\langle \text{right!}m \rangle) \quad (4)$$

Figure 7. Temporal logic specification of an unbounded buffer.

CSP

$BUFFER = P_{<>}$
where $P_{<>} = \text{left?}m \rightarrow P_{<m>}$
and $P_{<m>\wedge s} = (\text{left?}n \rightarrow P_{<m>\wedge s\wedge <n>} \mid \text{right!}m \rightarrow Ps)$

BUFFER **sat** (right ≤ left) ∧ (**if** right = left **then** left ∉ *ref* **else** right ∈ ref)

Figure 8. CSP program and specification of an unbounded buffer.

Transition Axioms

module BUFFER **with subroutines** PUT, GET

state functions:

buffer : **sequence of** message
parg : *message* **or** *NULL*
gval : *message* **or** *NULL*

initial conditions:

$|buffer| = 0$

safety properties

1. (a) *at*(PUT) ⇒ *parg* = PUT.PAR
   (b) *after*(PUT) ⇒ *parg* = *NULL*
2. (a) *at*(GET) ⇒ *gval* = *NULL*
   (b) *after*(GET) ⇒ GET.PAR = *gval*
3. **allowed changes to** *buffer*
   parg **when** *in*(PUT)
   gval **when** *in*(GET)
   (a) α[BUFFER]:*in*(PUT) ∧ *parg* ≠ *NULL* →
   parg′ = *NULL* ∧ *buffer′* = *buffer* ∗ *parg*
   (b) α[BUFFER]:*in*(GET) ∧ *gval* = *NULL* ∧ |*buffer*| > 0 →
   gval′ ≠ *NULL* ∧ *buffer* = *gval′* ∗ *buffer′*

liveness properties

4. *in*(PUT) ∧ |*buffer*| < *min* ⇝ *after*(PUT)
5. *in*(GET) ∧ |*buffer*| > 0 ⇝ *after*(GET)

Figure 9. Transition axiom specification of an unbounded buffer.

Concurrency: Temporal Logic, CSP, Transition Axioms

	Temporal Logic (1980)	**CSP** (1985)	**Transition Axioms** (1983)
Base	Property-oriented	Model-oriented (for specifying) Property-oriented (for proving)	Model-oriented (for specifying) Property-oriented (for proving)
Readability	Normal	Normal	Good
Specifiability	Bad	Bad	Good
Size	Compact	Compact	Long
Tool-Support	Many related tools	Proof Checker B	N/A

Bounds of Formal Methods

Between the Ideal and Real Worlds
Assumptions about the Environment

Between the Ideal and Real Worlds

- Formal methods are
  - Based on mathematics
  - But not entirely mathematical

- Two important boundaries between the mathematical and the real world

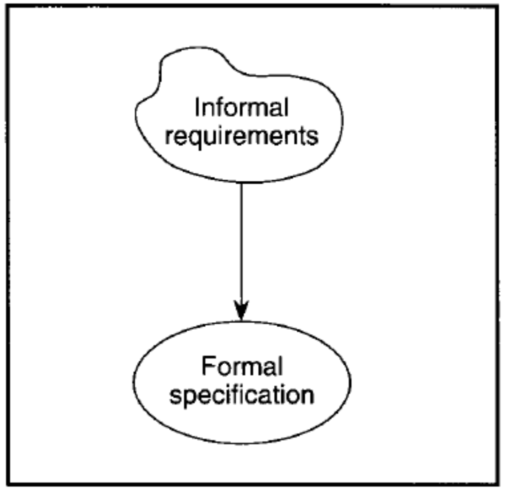


Figure 10. Mapping informal requirements for a formal specification.

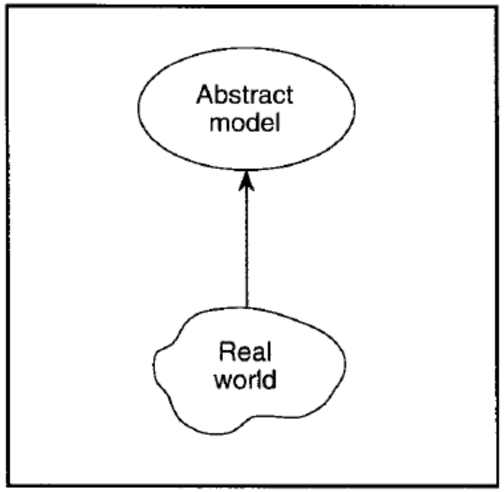


Figure 11. Mapping the real world to an abstract model.

Assumptions about the Environment

- There is a boundary between a real system and its environment
  - Environment is out of the scope of formal specifications (Open System)
  - Except, Gist specification language
    - `Environment` $\Rightarrow$ `System`
    - `Environment` is a set of assumptions
    - `System` is a set of constraints on its behaviors placed by specifiers
  - Implicit assumptions in programming language areas
  - Specifiers should make explicit as many assumptions as possible.

- Hazard Analysis
  - Identify a system's safety-critical components
    - FTA, FMEA, HAZOP
  - A complementary technique to formal methods

Concluding Remarks

Formal Methods
Challenges

Formal Methods

- In a strict mathematical sense,
  - Formal methods differ greatly from one another
- In a practical sense,
  - Formal methods do not differ radically from one another

- Formal methods can be used
  1. Identify
     - Deficiencies in informal requirements
     - Discrepancies between a specification and an implementation
     - Errors in existing programs and systems
  2. Specify
     - Medium-sized and nontrivial problems
     - Functional behavior
  3. Provide
     - Deeper understanding of the behavior of systems

Challenges

1. Specifying nonfunctional behavior
   – Reliability, safety, real-time, performance, human factors
2. Combining different methods
   – Domain specific + General
   – Formal + Informal
3. Building more usable and robust tools
   – Can manage large specifications
   – Can perform more complicated semantic analysis
4. Building specification libraries
   – Reuse in general or domain-specific purpose
5. Formal methods based software development
6. Scale up existing techniques
7. Educating and training